Classroom Worksheet-2 IIT-JEE ADVANCED

The answer to each of the questions are upto 4 digits. (Integer Type)

1. The normal at the point $P\left(2,\frac{1}{2}\right)$ on the curve $xy = 1$, meets the curve again Q. If m is the slop of the curve at Q, then find |m|.

2. If the range of real values of b for which the equation $(x^4 + 4x^2 + 4) - (b + 4)(x^4 + 6x^2 + 8) - (b + 5)(x^4 + 8x^2 + 16) = 0$ has atleast one real solution is $[\alpha,\beta)$ then find the value of $(2\alpha - 5\beta)$.

3. Let $A = \sin^{-1}(\sin 8) - \tan^{-1}(\tan 10) + \cos^{-1}(\cos 12) - \sec^{-1}(\sec 9) + \cot^{-1}(\cot 6) - \text{cosec}^{-1}(\text{cosec } 7)$ and $B = \sin^{-1}\left(\sin\frac{33\pi}{7}\right) + \cos^{-1}\left(\cos\frac{46\pi}{7}\right) + \tan^{-1}\left(-\tan\frac{13\pi}{8}\right) + \cot^{-1}\left(\cot\left(\frac{-19\pi}{8}\right)\right)$.
Find the value of (7B – A).

4. Consider the ellipse $3x^2 + y^2 = 1$. If range of $x^2 + 2xy + 2y^2$ is $[a, b]$, then find the value of $6(a + b)$.

5. Let a, b, c are distinct integers and w, w^2 are the imaginary cube roots of unity. If minimum value of $\left|a + bw + cw^2\right| + \left|a + bw^2 + cw\right|$ is $n^{\frac{1}{4}}$ where $n \in N$ then find the value of n.

6. If α & β are roots of the equation $\begin{bmatrix}1 & 25\end{bmatrix}\begin{bmatrix}0 & \frac{1}{2}\\ -1 & \frac{1}{2}\end{bmatrix}^5\begin{bmatrix}1 & -1\\ 2 & 0\end{bmatrix}^{10}\begin{bmatrix}0 & \frac{1}{2}\\ -1 & \frac{1}{2}\end{bmatrix}^5\begin{bmatrix}x^2 - 5x + 20\\ x + 2\end{bmatrix} = \begin{bmatrix}40\end{bmatrix}$ then find the value of
$(1 - \alpha)(1 - \beta)$.

7. If abscissa and ordinate of vertex of parabola $y^2 + 2ax + 2by - 1 = 0 (a > 0)$ is same, then find the least value of L^2(L is the length of latus rectum.)

8. A plane P passes through the point (−1, 2, 1) and contains the line of intersection of the planes $x + y - z = 0$ and $2x - y + 3z - 1 = 0$. Let plane intersects the coordinate axes at A, B, C and O is the origin. If V be the volume of the tetrahedron OABC which can be expressed as rational number in the form of $\frac{m}{n}$ (m, n $\in$ N), then find the smallest value of (m + n).

9. If $a_1, a_2, a_3, \ldots\ldots$, is an arithmetic progression with common difference unity and $a + (a + 1) + (a + 2) + \ldots\ldots\ldots + (a + 97) = 137$ then find the value of $\sum_{k=1}^{49} 2_{2k}$

10. Find the number of triplets (x, y, z) where $x, y, z = [0, 2\pi]$ satisfying the inequality $(4 + \sin 4x)(2 + \cot^2 y)(1 + \sin^4 z) \le 12 \sin^2 z$.

11. Two sequences G(n) and P(n) , n $\in$ N, are defined as

$$G(n) = x^n + x^{n-1} + x^{n-2} + \ldots\ldots + x^2 + x$$

$$P(n) = \frac{1}{x^n} + \frac{1}{x^{n-1}} + \frac{1}{x^{n-2}} + \ldots\ldots + \frac{1}{x^2} + \frac{1}{x}$$

If x satisfies the equation $x^2 - x + 1 = 0$ and Im(x) < 0 then find the absolute value of G(11) + P(11).

12. Suppose $\vec{V_1} = \hat{i} + \hat{j} - 2\hat{k}$,$\vec{V_2} = \hat{i} - 2\hat{j} + \hat{k}$ & $\vec{V_3} = -2\hat{i} + 2\hat{j} + \hat{k}$ are three vectors. Let $\vec{V}$ be a vector such that it can be expressed as a linear combination of $\vec{V_1}$ & $\vec{V_2}$. Also $\vec{V} \cdot \vec{V_3} = 0$ and the projection of the vector $\vec{V}$ on $\hat{i} - \hat{j} + \hat{k}$ is $6\sqrt{3}$. If $\vec{V} = \lambda\left(\hat{i} + 3\hat{j} - 4\hat{k}\right)$ then find the absolute value of λ.

13. If t is real and $\lambda = \dfrac{t^2 - 3t + 4}{t^2 + 3t + 4}$, then find number of solutions of the system of equations $3x - y + 4z = 3$, $x + 2y - 3z = -2$, $6x + 5y + \lambda z = -3$ for a particular value of λ.

14. Suppose $f : R \to R^4$ and $g : R \to R^4$ are differentiable functions such that $x \cdot g(f(x)) \cdot f'(g(x)) \cdot g'(x) = f(g(x)) \cdot g'(f(x)) \cdot f'(x)$ for all real x. Further $\int_0^a f(g(x))dx = 1 - \dfrac{e^{-2a}}{2}$ for all a ∈ R. Given that $g(f(0)) = 1$. If $g(f(4)) = e^{-k}$, then find k.

15. Let a, b, c be in geometric progression and $a - b$, $c - a$, $b - c$ are in harmonic progression. Find the number of integral values of b in interval [−50,100] so that both roots of the quadratic equation $(a + c)x^2 + bx + 4b^2 = 0$ are positive.

16. Let a, b and c be the side lengths of a triangle ABC and assume that $a \le b$ and $a \le c$. If the $x = \dfrac{b + c - a}{2}$ then find the minimum value of $\dfrac{ax}{rR}$, where r and R denotes in radius and circumradius of triangle ABC.

17. Let g be a differentiable function on R and satisfies $g(x) = x^2 + \int_0^x e^{-1} g(x - t)dt$. Find the value of definite integral $\int_{-3}^{3} g(x)dx$.

18. TP and TQ are tangents to parabola $y^2 = 4x$ at P and Q, where T is any point on $y^2 = 4(x + 1)$. If the locus of the middle point of chord PQ is parabola whose length of latus rectum is L, then find the value of (50) L.

19. Let f be a real valued function defined at R such that one root of the quadratic equation $x^2 - 3tx + 2f(t) + f(2 - 1) = 0$ is double the other root, then find $f'(7)$.

20. Find the smallest integral value of a such that $|x + a - 3| + |x - 2a| = |2x - a - 3|$ is true $\forall x \in R$